

LÖSUNGEN

$$7. \quad y = \frac{x(x-8)^2(x+8)^2}{2000} = \frac{x^5 - 128x^3 + 4096x}{2000}$$

$$y' = \frac{5x^4 - 384x^2 + 4096}{2000}$$

$$y'' = \frac{20x^3 - 768x}{2000} = \frac{x(20x^2 - 768)}{2000}$$

Graph unten links

Nullstellen: $y = 0$

$$\frac{x(x-8)^2(x+8)^2}{2000} = 0$$

x	y	y'
0	0	2
±8	0	0

Extrema: $y' = 0$

$$\frac{5x^4 - 384x^2 + 4096}{2000} = 0$$

x	y	y'
±8	0	0
±3.6	±4.7	0

Wendestellen: $y'' = 0$

$$\frac{x(20x^2 - 768)}{2000} = 0$$

x	y	y'
0	0	0
±6.2	±2	-1.6

Zusatzpunkt

x	y	y'
10	6.5	7.8

$$x \rightarrow \pm\infty \Rightarrow y \rightarrow \pm\infty$$

$$2. \quad y = \frac{1}{10}(x^4 - 20x^2 + 64) = \frac{x^4 - 20x^2 + 64}{10}$$

$$y' = \frac{4x^3 - 40x}{10} = \frac{4x(x^2 - 10)}{10}$$

$$y'' = \frac{12x^2 - 40}{10} = \frac{4(3x^2 - 10)}{10}$$

Graph Mitte unten

Nullstellen: $y = 0$

$$\frac{x^4 - 20x^2 + 64}{10} = 0$$

$$(x^2 - 16)(x^2 - 4) = 0$$

x	y	$y' = \frac{4x}{10}(x^2 - 10)$
±4	0	±9.6
±2	0	∓4.8

Extrema: $y' = 0$

$$\frac{4x(x^2 - 10)}{10} = 0$$

x	y	y'
0	6.4	0
±√10	-3.6	0

Wendestellen: $y'' = 0$

$$\frac{4(3x^2 - 10)}{10} = 0$$

x	y	$y' = \frac{4x}{10}(x^2 - 10)$
±1.8	0.84	∓5

$$x \rightarrow \pm\infty \Rightarrow y \rightarrow +\infty$$

3.
$$y = 2x^2 - \frac{x^4}{4} = \frac{8x^2 - x^4}{4} = \frac{x^2(8 - x^2)}{4}$$

$$y' = 4x - x^3 = x(4 - x^2)$$

$$y'' = 4 - 3x^2$$

Graph oben links

Nullstellen: $y = 0$

$$\frac{x^2(8 - x^2)}{4} = 0$$

x	y	$y' = \frac{4x}{10}(x^2 - 10)$
0	0	0
$\pm\sqrt{8}$	0	∓ 11.3

Extrema: $y' = 0$

$$x(4 - x^2) = 0$$

x	y	$y' = \frac{4x}{10}(x^2 - 10)$
0	0	0
± 2	4	0

Wendestellen: $y'' = 0$

$$4 - 3x^2 = 0$$

x	y	$y' = \frac{4x}{10}(x^2 - 10)$
± 1.15	2.2	± 3.1

$x \rightarrow \pm \infty \Rightarrow y \rightarrow -\infty$

6.
$$y = \frac{x^2}{300}(3x^2 - 40x + 150) = \frac{3x^4 - 40x^3 + 150x^2}{300}$$

$$y' = \frac{12x^3 - 120x^2 + 300x}{300} = \frac{4x(x^2 - 10x + 25)}{100} = \frac{4x(x - 5)^2}{100}$$

$$y'' = \frac{36x^2 - 240x + 300}{300} = \frac{3x^2 - 20x + 25}{25}$$

Graph Mitte rechts

Nullstellen: $y = 0$

$$\frac{x^2}{300}(3x^2 - 40x + 150) = 0$$

x	y	y'
0	0	0

Extrema: $y' = 0$

$$\frac{4x(x - 5)^2}{100} = 0$$

x	y	y'
0	0	0
5	2.1	0

Wendestellen: $y'' = 0$

$$\frac{3x^2 - 20x + 25}{25} = 0$$

x	y	y'
5	2.1	0
1.7	0.85	0.74

Einige zusätzliche Punkte

x	y	y'
-2	3.2	-4
8	4.7	3

$x \rightarrow \pm \infty \Rightarrow y \rightarrow +\infty$

$$5. \quad y = \frac{(x-4)^2(x+4)}{40} = \frac{x^3 - 4x^2 - 16x + 64}{40}$$

Graph oben rechts

$$y' = \frac{3x^2 - 8x - 16}{40}$$

$$y'' = \frac{6x - 8}{40} = \frac{3x - 4}{20}$$

Nullstellen: $y = 0$

$$\frac{(x-4)^2(x+4)}{40} = 0$$

x	y	y'
± 4	0	1.6

Extrema: $y' = 0$

$$\frac{3x^2 - 8x - 16}{40} = 0$$

x	y	y'
4	0	0
$\frac{4}{3}$	1.9	0

Wendestellen: $y'' = 0$

$$\frac{3x - 4}{20} = 0$$

x	y	y'
$\frac{4}{3}$	0.95	-0.5

$$x \rightarrow \pm \infty \Rightarrow y \rightarrow \pm \infty$$

$$4. \quad y = \frac{x^2}{12}(x^2 - 2x - 12) = \frac{x^4 - 2x^3 - 12x^2}{12}$$

Graph Mitte oben

$$y' = \frac{4x^3 - 6x^2 - 24x}{12} = \frac{x(2x^2 - 3x - 12)}{6}$$

$$y'' = \frac{12x^2 - 12x - 24}{12} = x^2 - x - 2 = (x-2)(x+1)$$

Nullstellen: $y = 0$

$$\frac{x^4 - 2x^3 - 12x^2}{12} = 0$$

x	y	y'
0	0	0
4.6	0	13
-2.6	0	-4

Extrema: $y' = 0$

$$\frac{x(2x^2 - 3x - 12)}{6} = 0$$

x	y	y'
0	0	0
3.3	-7	0
-1.8	-1.4	0

Wendestellen: $y'' = 0$

$$(x-2)(x+1) = 0$$

x	y	y'
2	-4	-3.3
-1	-0.75	1.2

$$x \rightarrow \pm \infty \Rightarrow y \rightarrow +\infty$$