21. $f(x) = \frac{1}{6}x^3 - \frac{7}{8}x^2 - x = \frac{x}{24}(4x^2 - 21x - 24)$

$f'(x) = \frac{1}{2}x^2 - \frac{7}{4}x - 1 = \frac{1}{4}(2x^2 - 7x - 4)$

$f''(x) = x - \frac{7}{4}$

Keine erkennbare Symmetrie

$x \to \pm\infty \quad \Rightarrow \quad y \to \pm\infty$

Nullstellen: $x = -0.965$

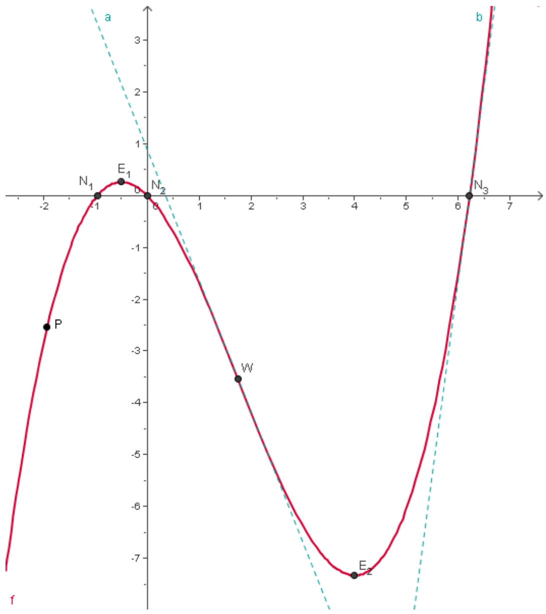
$x = 0$

$x = 6.215 \qquad f'(6.215) \approx 7.4$

Maximum: $\left(-\frac{1}{2}\middle|\frac{25}{96}\right)$

Minimum: $\left(4\middle|-\frac{22}{3}\right)$

Wendepunkt: $\left(\frac{7}{4}\middle|-\frac{679}{192}\right) \qquad f'\left(\frac{7}{4}\right) \approx -2.5$



22. $f(x) = \frac{1}{4}x^4 - 2x^2 = \frac{1}{4}x^2(x^2 - 8)$

$f'(x) = x^3 - 4x = x(x^2 - 4)$

$f''(x) = 3x^2 - 4$

Symmetrisch zur y-Achse

$x \to \pm\infty \quad \Rightarrow \quad y \to +\infty$

Nullstellen: $N_1\ (-2\sqrt{2}|0) \qquad f'(-2\sqrt{2}) = -8\sqrt{2}$

$N_2\ (0|0)$

$N_3\ (2\sqrt{2}|0) \qquad f'(2\sqrt{2}) = 8\sqrt{2}$

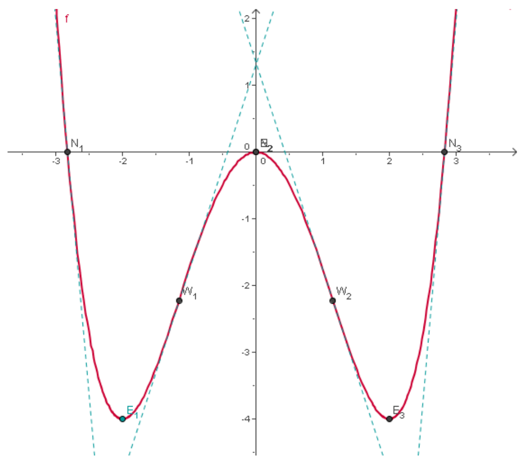
Extrema: $E_1\ (-2|-4)$ Min

$E_2\ (0|0)$ Max

$E_3\ (2|4)$ Min

Wendepkte: $W_1\ \left(\left(-\frac{2}{\sqrt{3}}\middle|0\right)\right) \qquad f'\left(-\frac{2}{\sqrt{3}}\right) \approx 1.15$

$W_2\ \left(\left(+\frac{2}{\sqrt{3}}\middle|0\right)\right) \qquad f'\left(\frac{2}{\sqrt{3}}\right) \approx -1.15$



23. $f(x) = \frac{1}{8}x^4 + 4x = \frac{1}{8}x(x^3 + 32)$

$f'(x) = \frac{1}{2}x^3 + 4 = \frac{1}{2}(x^3 + 8)$

$f''(x) = \frac{3}{2}x^2$

Keine erkennbare Symmetrie

$x \to \pm\infty \quad \Rightarrow \quad y \to -\infty$

Nullstellen: N_1 $(\left(-\sqrt[3]{32}\,|\,0\right)$ $\quad f'\left(-\sqrt[3]{32}\right) = -12$

$N_2\ (0|0) \qquad f'(0) = 4$

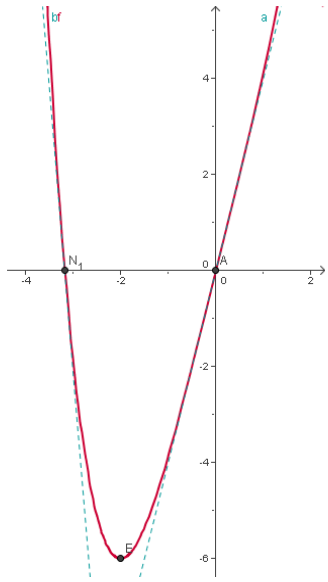
Minimum: $E(-2|-6)$

Die Kurve hat keine Wendepunkte:

vorher: $f''(-1) < 0$

nachher: $f''(1) < 0$

kein Wechsel der Krümmung



24. $f(x) = -\frac{1}{40}x^5 + 2x = -\frac{1}{40}x(x^4 - 80)$

$f'(x) = -\frac{1}{8}x^4 + 2 = -\frac{1}{8}(x^4 - 16)$

$f''(x) = -\frac{1}{2}x^3$

Symmetrisch zum Ursprung

$x \to \pm\infty \quad \Rightarrow \quad y \to m\infty$

Nullstellen: $N_1\left(-2\sqrt[4]{5}\,|\,0\right) \qquad f'\left(-2\sqrt[4]{5}\right) = -8$

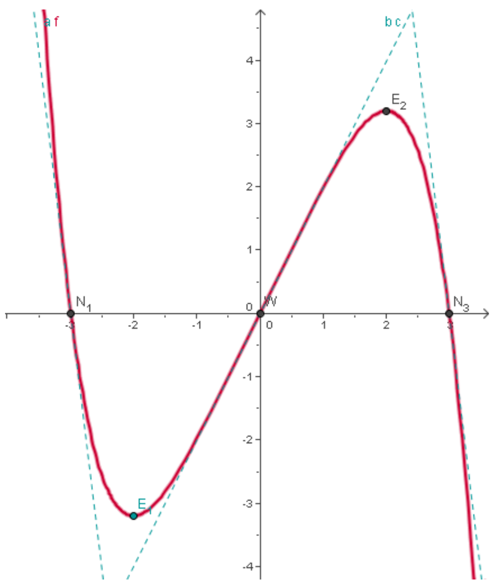
$N_2\ (0|0) \qquad f'(0) = 2$

$N_3\left(2\sqrt[4]{5}\,|\,0\right) \qquad f'\left(2\sqrt[4]{5}\right) = -8$

Extrema: $E_1\ (-2|3.2)$ Min

$E_2\ (2|3.2)$ Max

Wendepkte: $W_1 = N_1$



25 $f(x) = \frac{1}{20}x^5 - \frac{1}{4}x^4 = \frac{1}{20}x^4(x-9)$

$f'(x) = \frac{1}{4}x^4 - x^3 = \frac{1}{4}x^3(x-4)$

$f''(x) = x^3 - 3x^2 = x^2(x-3)$

keine erkennbare Symmetrie

$x \to \pm\infty \quad \Rightarrow \quad y \to \pm\infty$

Nullstellen: $N_1\,(0|0)$ $\quad f'(0) = 0$

$N_2\,(5|0)$ $\quad f'(5) = 31.25$

Extrema: Max $(0|0)$

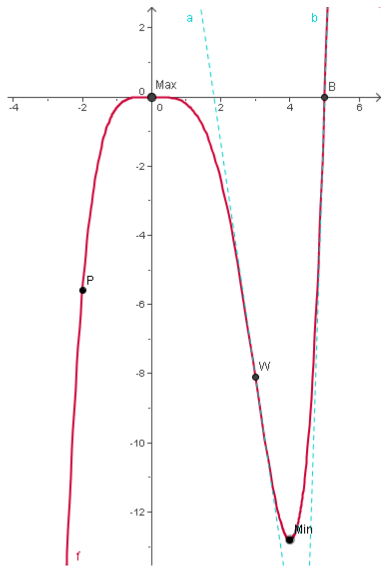
Min $(4|-12.8)$

Wendepkte: $W\,(3|-8.1)$ $\quad f'(3) = -6.75$

$(0|0)$ ist kein Wendepunkt:

vorher: $f'(-1) > 0$ steigend

nachher: $f'(1) < 0$ fallend



26. $f(x) = \frac{1}{6}x^3 + 2x^2 + 6x$

$= \frac{1}{6}x\,(x^2 + 12x + 36) = \frac{1}{6}x\,(x+6)^2$

$f'(x) = \frac{1}{2}x^2 + 4x + 6$

$= \frac{1}{2}(x^2 + 8x + 12) = \frac{1}{2}(x+6)(x+2)$

$f''(x) = x + 4$

keine erkennbare Symmetrie

$x \to \pm\infty \quad \Rightarrow \quad y \to \pm\infty$

Nullstellen: $N_1\,(-6|0)$ $\quad f'(-6) = 0$

$N_2\,(0|0)$ $\quad f'(0) = 6$

Extrema: Max $(-6|0)$

Min $(-2|-\frac{16}{3})$

Wendepkte: $W\,(-4|-\frac{8}{3})$ $\quad f'(-4) = -4$