1. $x^4 - 15x^2 - 16 = (x^2 - 16)(x^2 + 1) = (x - 4)(x + 4)(x^2 + 1) = 0 \quad \Rightarrow \quad \mathbf{x_{1,2} = \pm 4}$

2. $x^5 - 17x^3 + 16x = x(x^4 - 17x^2 + 16) = x(x^2 - 1)(x^2 - 16) = x(x - 1)(x + 1)(x - 4)(x + 4) = 0$

$\mathbf{L = \{ 0, \pm 1, \pm 4 \}}$

3. $x^3 - \frac{8}{x^3} = 7 \quad \Big| \cdot x^3$

$x^6 - 8 = 7x^3 \quad \Rightarrow \quad x^6 - 7x^3 - 8 = (x^3 - 8)(x^3 + 1) = 0$ aus $x^3 = 8 \quad \Rightarrow \quad \mathbf{x = 2}$

$x^3 = -1 \quad \Rightarrow \quad \mathbf{x = -1}$

4. $\sqrt{x^2 + 9} + x^2 - 21 = 0$

$\sqrt{x^2 + 9} = 21 - x^2 \quad \Big| \; ()^2$

$x^2 + 9 = 441 - 42x^2 + x^4$

$0 = x^4 - 43x^2 + 432$

TR liefert: $x^2 = 27$ Probe: $\sqrt{27 + 9} + 27 - 21 = 6 + 27 - 21 \neq 0$

$x^2 = 16$ $\sqrt{16 + 9} + 16 - 21 = 5 + 16 - 21 = 0 \quad \Rightarrow \quad \mathbf{x_{1,2} = \pm 4}$

5. $\sqrt{x + 12} = 6 - \sqrt{x} \quad \Big| \; ()^2$

$x + 12 = 36 - 12\sqrt{x} + x$

$12\sqrt{x} = 24$

$\sqrt{x} = 2$

$\mathbf{x = 4}$ Probe: $\sqrt{4 + 12} = 4 = 6 - \sqrt{4}$ stimmt!

6. $\sqrt{2 - x} + \sqrt{3 - x} = \sqrt{5 - 2x} \quad \Big| \; ()^2$

$(2 - x) + 2\sqrt{(2 - x)(3 - x)} + (3 - x) = 5 - 2x$

$2\sqrt{(2 - x)(3 - x)} = 0$

$\sqrt{(2 - x)(3 - x)} = 0 \quad \Big| \; ()^2$

$(2 - x)(3 - x) = 0$

Für $x = 3$ ist die erste Wurzel nicht definiert;

Einzige Lösung: $\mathbf{x = 2}$ Probe: $\sqrt{0} + \sqrt{1} = \sqrt{1}$