Berechnen Sie die bestimmten Integrale:

$$\int_0^{\frac{\pi}{2}} \cos x \sin^2 x dx \qquad \int_0^{\frac{\pi}{2}} \sin x \cos^2 x \, dx \qquad \int_0^{\frac{\pi}{2}} \sin x \cos x \, dx \qquad \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1+\sin^2 x} \, dx$$

[TSME 1990, Teilaufgabe]

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a) $$\int_0^{\frac{\pi}{2}} \cos x \sin^2 x dx = \int u^2 du = \left[\frac{u^3}{3}\right] = \left[\frac{\sin^3 x}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{3}$$

Substitution! $\sin x = u$

$\cos x dx = du$

b) $$\int_0^{\frac{\pi}{2}} \sin x \cos^2 x dx = \int -u^2 du = \left[-\frac{u^3}{3}\right] = \left[-\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}} = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$

Substitution! $\cos x = u$

$-\sin x \, dx = du$

c) $$\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx = \int u \, du = \left[\frac{u^2}{2}\right] = \left[\frac{\sin^2 x}{2}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}$$

Substitution! $\sin x = u$

$\cos x \, dx = du$ ($\cos x = u$ wäre auch möglich)

d) $$\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1+\sin^2 x} \, dx = \int \frac{1}{2} \cdot \frac{1}{u} du = \left[\frac{\ln u}{2}\right] = \left[\frac{\ln(1+\sin^2 x)}{2}\right]_0^{\frac{\pi}{2}} = \frac{\ln 2 - \ln 1}{2} = \frac{\ln 2}{2}$$

Substitution! $1 + \sin^2 x = u$

$2 \sin x \cdot \cos x \, dx = du$