

[Matur TSME, 1997, Flü

Gegeben ist die Funktion  $y = (\sin x)^2 + \sin x$

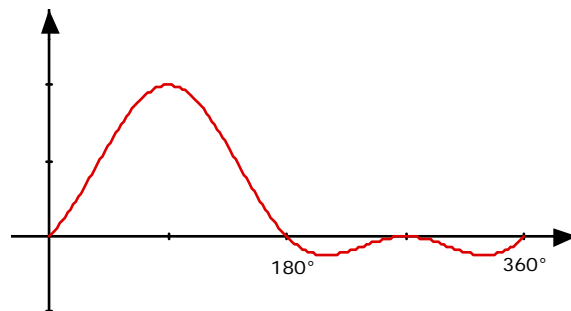
- a) Berechnen Sie Nullstellen, Extrema, Wendepunkte und zeichnen Sie den Graphen im Intervall  $[0; 2\pi]$   
 b) Berechnen Sie die Fläche zwischen Kurve und x-Achse im Bereich  $[0; \pi]$  (exakt!)

Nullstellen:  $y = (\sin x)^2 + \sin x = \sin x \cdot (\sin x + 1) = 0$       $\sin x = 0 \Rightarrow x = 0^\circ, 180^\circ, 360^\circ$   
 $\sin x = -1 \Rightarrow x = 270^\circ$

Extrema:  $y' = 2 \sin x \cos x + \cos x = \cos x \cdot (2 \sin x + 1) = 0$       $\cos x = 0 \Rightarrow x = 90^\circ, 270^\circ$   
 $\sin x = -\frac{1}{2} \Rightarrow x = 210^\circ, 330^\circ$

Wendepkt:  $y'' = 2(\cos^2 x - \sin^2 x) - \sin x$       $\sin x = -0.843 \Rightarrow x = 302.5^\circ, 237.5^\circ$   
 $= 2(1 - \sin^2 x - \sin^2 x) - \sin x$       $\sin x = 0.593 \Rightarrow x = 36.4^\circ, 143.6^\circ$   
 $= -4 \sin^2 x - \sin x + 2 = 0$

x	y
0	0
$180^\circ = \pi$	0
$360^\circ = 2\pi$	0
$270^\circ = 1.5\pi$	0
$90^\circ = 0.5\pi$	2
$270^\circ = \frac{7\pi}{6}$	-0.25
$330^\circ = \frac{11\pi}{6}$	-0.25
$302.5^\circ = 5.28$	-0.13
$237.5^\circ = 4.14$	-0.13
$36.4^\circ = 0.63$	0.94
$143.6^\circ = 2.51$	0.94



$$A = \int_0^{\pi} (\sin^2 x + \sin x) dx = \left[ \frac{x}{2} - \frac{1}{2} \sin x \cos x - \cos x \right]_0^{\pi} = \left( \frac{\pi}{2} - 0 + 1 \right) - (0 - 0 - 1) = 2 + \frac{\pi}{2}$$