Die Aufgaben

1 $\left(\frac{1}{r-s}-\frac{1}{r+s}\right)^2=$

2 $\frac{x^4}{46y^3}:\left(\frac{x^2}{23y}:\frac{5y}{x^2}\right)=$

3 $\frac{a^2-13a-230}{10}:\left(\frac{a}{100}+\frac{1}{10}\right)=$

4 $\left(\frac{ab}{a-b}+a\right)\left(\frac{ab}{a+b}-b\right)\cdot\frac{b-a}{ab^2}=$

5 $\frac{3a}{3a-2b}\cdot\frac{3a}{2b}-\left(\frac{3a}{3a-2b}+\frac{3a}{2b}\right)=$

6 $\left(\frac{8x^2+4x+1}{4x^2-2x}-\frac{2x}{2x-1}\right)\frac{6x-3}{4x^2+2x}=$

7 $\left(\frac{n^3-2n-1}{n^2-1}-n\right)\left(n-\frac{2n^2}{n+1}\right)=$

8 $\left(x-\frac{1}{x}\right):\left(x+\frac{1}{x}\right)=$

9 $\frac{72(a-b)^2}{25a^3}\left(\frac{54c^3}{(a-b)^3}:\frac{81c^2}{5a^2}\right)=$

10 $\left(\frac{2a+1}{2a-1}-\frac{2a-1}{2a+1}\right)\cdot\left(\frac{a}{2}-\frac{1}{2}+\frac{1}{8a}\right)=$

11 $\left(\frac{a^3-1}{a^3}-\frac{a^2-a-1}{a^2}-\frac{1}{a}\right):\frac{1}{a^3}=$

12 $\frac{a^2b}{8}\left\{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{c}\right)\left(\frac{1}{b}-\frac{1}{a}\right)\right\}=$

1 $\left(\frac{1}{r-s}-\frac{1}{r+s}\right)^2=\left(\frac{r+s-(r-s)}{(r-s)(r+s)}\right)^2=\left(\frac{2s}{r^2-s^2}\right)^2=\frac{4s^2}{(r^2-s^2)^2}$

Ich beginne hier mit der Subtraktion, weil das Resultat relativ einfach ist.

2 $\frac{x^4}{46y^3}:\left(\frac{x^2}{23y}:\frac{5y}{x^2}\right)=\frac{x^4}{46y^3}:\left(\frac{x^2}{23y}\cdot\frac{x^2}{5y}\right)=\frac{x^4}{46y^3}:\frac{x^4}{5\cdot 23y^2}=\frac{x^4}{46y^3}\cdot\frac{5\cdot 23y^2}{x^4}=\frac{5}{2y}$

Klammerrechnung zuerst!

3 $\frac{a^2-13a-230}{10}:\left(\frac{a}{100}+\frac{1}{10}\right)=\frac{(a-23)(a+10)}{10}:\frac{a+10}{100}=\frac{(a-23)(a+10)}{10}\cdot\frac{100}{a+10}=10\,(a-23)$

4 $\left(\frac{ab}{a-b}+a\right)\left(\frac{ab}{a+b}-b\right)\cdot\frac{b-a}{ab^2}=\left(\frac{ab+a(a-b)}{a-b}\right)\left(\frac{ab-b\,(a+b)}{a+b}\right)\cdot\frac{b-a}{ab^2}$

$=\frac{a^2}{a-b}\cdot\frac{-b^2}{a+b}\cdot\frac{b-a}{ab^2}=\frac{-a\,(b-a)}{(a-b)(a+b)}=\frac{a\,(-b+a)}{(a-b)(a+b)}=\frac{a}{a+b}$

5 $\frac{3a}{3a-2b}\cdot\frac{3a}{2b}-\left(\frac{3a}{3a-2b}+\frac{3a}{2b}\right)=\frac{9a^2}{2b\,(3a-2b)}-\frac{6ab+3a\,(3a-2b)}{2b\,(3a-2b)}$

$=\frac{9a^2-(6ab+9a^2-6ab)}{2b\,(3a-2b)}=\frac{0}{2b\,(3a-2b)}=0$

6 $\left(\frac{8x^2+4x+1}{4x^2-2x}-\frac{2x}{2x-1}\right)\cdot\frac{6x-3}{4x^2+2x}=\left(\frac{8x^2+4x+1}{2x\,(2x-1)}-\frac{2x}{2x-1}\right)\cdot\frac{6x-3}{4x^2+2x}$

$=\frac{8x^2+4x+1-4x^2}{2x\,\cancel{(2x-1)}}\cdot\frac{3\,\cancel{(2x-1)}}{2x\,(2x+1)}=\frac{4x^2+4x+1}{2x}\cdot\frac{3}{2x\,(2x+1)}$

$=\frac{(2x+1)^2\cdot 3\,(x-2)}{4x^2\,(2x+1)}=\frac{3\,(2x+1)}{4x^2}$

7 $$\left(\frac{n^3-2n-1}{n^2-1}-n\right)\left(n-\frac{2n^2}{n+1}\right)=\frac{n^3-2n-1-n(n^2-1)}{n^2-1}\cdot\frac{n(n+1)-2n^2}{n+1}$$

$$=\frac{n^3-2n-1-n^3+n}{n^2-1}\cdot\frac{n^2+n-2n^2}{n+1}=\frac{-n-1}{n^2-1}\cdot\frac{n-n^2}{n+1}$$

$$=\frac{-1\cancel{(n+1)}\cdot(-n)\cancel{(-1+n)}}{\cancel{(n-1)}(n+1)\cdot\cancel{(n+1)}}=\frac{n}{n+1}$$

8 $$\left(x-\frac{1}{x}\right):\left(x+\frac{1}{x}\right)=\frac{x^2-1}{x}:\frac{x^2+1}{x}=\frac{x^2-1}{x}\cdot\frac{x}{x^2+1}=\frac{x^2-1}{x^2+1}$$

Divisionen durch eine Summe müssen meist so gelöst werden!

9 $$\frac{72(a-b)^2}{25a^3}\left(\frac{54c^3}{(a-b)^3}:\frac{81c^2}{5a^2}\right)=\frac{72(a-b)^2}{25a^3}\left(\frac{\cancel{54c^3}}{(a-b)^3}\cdot\frac{5a^2}{\cancel{81c^2}}\right)$$

$$=\frac{72(a-b)^2}{25a^3}\cdot\frac{2c\cdot 5a^2}{(a-b)^3\cdot 3}=\frac{48c}{5a(a-b)}$$

10 $$\left(\frac{2a+1}{2a-1}-\frac{2a-1}{2a+1}\right)\cdot\left(\frac{a}{2}-\frac{1}{2}+\frac{1}{8a}\right)=\frac{(2a+1)^2-(2a-1)^2}{(2a+1)(2a-1)}\cdot\frac{4a^2-4a+1}{8a}$$

$$=\frac{4a^2+4a+1-(4a^2-4a+1)}{(2a+1)(2a-1)}\cdot\frac{(2a-1)^2}{8a}$$

$$=\frac{8a}{(2a+1)(2a-1)}\cdot\frac{(2a-1)^2}{8a}=\frac{2a-1}{2a+1}$$

11 $$\left(\frac{a^3-1}{a^3}-\frac{a^2-a-1}{a^2}-\frac{1}{a}\right):\frac{1}{a^3}=a^3\cdot\left(\frac{a^3-1}{a^3}-\frac{a^2-a-1}{a^2}-\frac{1}{a}\right)$$

$$=(a^3-1)-a(a^2-a-1)-a^2=a^3-1-a^3+a^2+a-a^2=a-1$$

12 $$\frac{a^2b}{8}\left\{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{c}\right)\left(\frac{1}{b}-\frac{1}{a}\right)\right\}$$

$$=\frac{a^2b}{8}\left\{\frac{1}{a^2}-\frac{1}{ab}+\frac{1}{ac}-\frac{1}{ab}+\frac{1}{b^2}-\frac{1}{bc}+\frac{1}{b^2}-\frac{1}{ab}-\frac{1}{ab}+\frac{1}{a^2}+\frac{1}{bc}-\frac{1}{ac}\right\}$$

$$=\frac{a^2b}{8}\left\{\frac{2}{a^2}-\frac{4}{ab}+\frac{2}{b^2}\right\}=\frac{a^2b}{8}\cdot\frac{2b^2-4ab+2a^2}{a^2b^2}=\frac{2(b^2-2ab+a^2)}{8b}=\frac{(a-b)^2}{4b}$$