12 vermischte Aufgaben zum Rechnen mit Potenzen mit natürlichen Exponenten.

Vereinfachen:

1 $16 \cdot 5^{n-1} + 9 \cdot 5^{n-1} =$

2 $5 \cdot 2^{2n} + 4^{n} =$

3 $3 \cdot 36^{n} - 6^{2n+1} =$

Faktorisieren:

4 $x^{2n} - 9 =$

5 $a^{2n+2} - 1 =$

6 $a^{n+2} + 4a^{n+1} + 4a^{n} =$

Kürzen:

7 $\dfrac{x^5 - x^4}{x^5 - x^3} =$

8 $\dfrac{a^n - a^{n+1}}{a^n - a^{n-1}} =$

9 $\dfrac{a^{4p+1} + 2a^{2p+1}b^{p} + ab^{2p}}{a^{4p}b - b^{2p+1}} =$

10 $\dfrac{4^5 \cdot 25^6}{10^{11}} =$

11 $\dfrac{6^5 \cdot 35^4}{9 \cdot 14^4 \cdot 15^3} =$

12 $\dfrac{4(12a^3b^4x^2)^3}{27(4a^2b^3x)^4} =$

Vereinfachen:

1 $16 \cdot 5^{n-1} + 9 \cdot 5^{n-1} = 5^{n-1}(16+9) = 5^{n-1} \cdot 25 = 5^{n-1} \cdot 5^2 = 5^{n+1}$

2 $5 \cdot 2^{2n} + 4^n = 5 \cdot 2^{2n} + (2^2)^n = 5 \cdot 2^{2n} + 2^{2n} = 2^{2n}(5+1) = 6 \cdot 2^{2n}$

oder: $5 \cdot 2^{2n} + 4^n = 5 \cdot (2^2)^n + 4^n = 5 \cdot 4^n + 4^n = 4^n \cdot (5+1) = 6 \cdot 4^n$

3 $3 \cdot 36^n - 6^{2n+1} = 3 \cdot (6^2)^n - 6^{2n+1} = 3 \cdot 6^{2n} - 6^{2n+1} = 6^{2n} \cdot (3 - 6^1) = -3 \cdot 6^{2n}$

Faktorisieren:

4 $x^{2n} - 9 = (x^n + 3)(x^n - 3)$ denn: $x^{2n} = (x^n)^2$

5 $a^{2n+2} - 1 = (a^{n+1} + 1)(a^{n+1} - 1)$ denn: $a^{2n+2} = (a^{n+1})^2$

6 $a^{n+2} + 4a^{n+1} + 4a^n = a^n(a^2 + 4a + 4) = a^n(a+2)^2$

Kürzen:

7 $\frac{x^5 - x^4}{x^5 - x^3} = \frac{x^4(x-1)}{x^3(x^2-1)} = \frac{x(x-1)}{(x+1)(x-1)} = \frac{x}{x+1}$

8 $\frac{a^n - a^{n+1}}{a^n - a^{n-1}} = \frac{a^n(1-a)}{a^{n-1}(a-1)} = -a^1 = -a$

9 $\frac{a^{4p+1} + 2a^{2p+1}b^p + ab^{2p}}{a^{4p}b - b^{2p+1}} = \frac{a(a^{4p} + 2a^{2p}b^p + b^{2p})}{b(a^{4p} - b^{2p})} = \frac{a(a^{2p} + b^p)^2}{b(a^{2p} - b^p)(a^{2p} + b^p)} = \frac{a(a^{2p} + b^p)}{b(a^{2p} - b^p)}$

10 $\frac{4^5 \cdot 25^6}{10^{11}} = \frac{(2^2)^5 \cdot (5^2)^6}{(2 \cdot 5)^{11}} = \frac{2^{10} \cdot 5^{12}}{2^{11} \cdot 5^{11}} = \frac{5}{2}$

11 $\frac{6^5 \cdot 35^4}{9 \cdot 14^4 \cdot 15^3} = \frac{(2 \cdot 3)^5 \cdot (5 \cdot 7)^4}{3^2 \cdot (2 \cdot 7)^4 \cdot (3 \cdot 5)^3} = \frac{2^5 \cdot 3^5 \cdot 5^4 \cdot 7^4}{3^2 \cdot 2^4 \cdot 7^4 \cdot 3^3 \cdot 5^3} = 2 \cdot 5 = 10$

12 $\frac{4(12a^3b^4x^2)^3}{27(4a^2b^3x)^4} = \frac{2^2(2^2 \cdot 3a^3b^4x^2)^3}{3^3(2^2a^2b^3x)^4} = \frac{2^2 \cdot 2^6 \cdot 3^3 \cdot a^9 \cdot b^{12} \cdot x^6}{3^3 \cdot 2^8 \cdot a^8 \cdot b^{12} \cdot x^4} = ax^2$